BETWEEN WORDS AND ARTEFACTS

Implementing history in the math class from kindergarten to teachers training

Frédéric MÉTIN
University of Burgundy, ESPÉ & IREM, Dijon, France
frederic.metin01@u-bourgogne.fr

ABSTRACT

This article presents several ways to introduce a historical perspective into mathematical activities at school level. First highlighting usual difficulties encountered by beginners in implementing a historical approach to their mathematics sessions; it then suggests different modes of appropriation of a variety of sources, from original texts to artefacts described in these texts. For instance it revisits the use of counting tokens of the medieval period as well as two-colored tiles various combinations of which were studied by Sébastien Truchet in 1704. Going back to texts, it is finally suggested another use of old pages as support for marginal writings, allowing comments and appropriation of their contents.

1 Introduction

For the first time in France, history of mathematics becomes a significant part of the national high school curriculum in 2019. As the general introduction of the curriculum suggests: "It may be wise to enlighten the course with elements of historical or epistemological contexts. History can also be considered a fruitful source of problems clarifying the meaning of certain notions". Thus, many items are followed by examples of possible original texts that can be used in the classroom, along with mentions of famous mathematicians' biographies. My aim in this paper is to convince the readers of the relevance of an historical approach to mathematical notions from primary school to university, despite many obstacles faced by anyone using original sources in the classroom.

Sometimes we are led to use attractive anecdotes at the expense of rigor, in order to give a historical perspective to our course on mathematics. Then the usual aridity of our mathematical discourse gives way to a pleasant coloration. Isn't it worth this slight sacrifice? Historians themselves cope with an evolving historical truth. Even in mathematics, contemporary myths die hard. For instance, many stories about the golden ratio persist (Neveux, 1995), and the use of the so-called thirteen-knot-rope still prevails when medieval geometry is discussed. Between historical rigor and pedagogical benefits, I try to find my way focusing on objects, artefacts which can be proposed for study. I will first point out usual pitfalls faced by beginners, and then suggest several ways of using original sources in the classroom, balancing between historical texts and artefacts we can find in these texts.

2 Common pitfalls of the historical approach

The historical approach in mathematics is not common. It even presents several impediments, which are often put forward by the teachers who practiced it: time required to prepare activities that must often be completely created; lack of historical culture about the content to be taught; fear about additional difficulties for students who are not interested in these "useless" subjects: mathematics and history. But these impediments are often offset by a real light shed on students by the use of old resources. Moreover, reading original texts generates a change of view on mathematics, which is discovered much more human than it was thought.

For instance recalling the historical context of the invention of decimals helps to strengthen the link between mathematics and the real world, since Stevin addresses his *Thiende* (Stevin, 1585) to a whole range of crafts that are likely to use it: astrologers, surveyors, tapestry measurers, gaugers, and so on. But as a consequence of this link to the real world, reading the text of the *Thiende* raises questions outside the mathematical field as Peltier and Briand have shown in their study of Stevin's text (Briand & Peltier, 2003). But another problem arises from this study: the authors date the *Thiende* to 1582, while its first edition dates back to 1585. Despite deep bibliographic researches, I have never been able to find any 1582 edition of this book. The confusion may come from Georges Ifrah's *Universal history of numbers* (Ifrah, 1998), which mistakes the *Thiende* for Stevin's *Tafelen van Interest* (Stevin, 1582). This is a tiny detail. It shows however the difficulty of being perfectly rigorous, even for specialists of the subject. Let us raise some other difficulties posed by the practice of the historical approach.

2.1 *Words, words, words*: the dangers of storytelling

One of the most common uses of the history of mathematics in the classroom consists in humanizing the concepts through anecdotes from the great inventors' lives. More and more often, it is also done by showing and reproducing daily use of mathematical notions at various times by ordinary people, and not only geniuses. However, simple anecdotes out of context would not have a great impact, so it is essential to take time to contextualize the concept and to promote a multidisciplinary activity. This supposes a certain historical and epistemological culture for teachers as storytellers.

It is partly the reason why the new EFEC[1] Bachelor's degree of the University of Burgundy includes a teaching unit on history and epistemology of mathematics. In this unit, I privilege a narrative hook, a form of storytelling, in order to attract the attention of my second year students, most of whom have a very small scientific background[2]. For me, presenting history of mathematics as part of the history of humankind was the ideal means to give a general perspective on the notions of number, size, geometric shapes, as well as on measuring and solving problems.

But the reception of these great stories by my audience did not comply with my objectives. Most students do not have a solid knowledge in both mathematics and history, and the wealth of information provided during the sessions was likely to blur the overall message. A revealer of the difficulties associated with this "grand narrative" approach is obviously the moment of evaluation. Let us give a typical example, which shows how vain it may be to imagine that a great saga in the manner of *the Lord of the Rings* facilitates the mastery of mathematical notions.

One of the questions of an MCQ test was: *Who invented algebra, when, and in which geographic area?* I admit now that these questions were ambiguous, but I thought at that moment that I could gather unequivocal answers. To the contrary, the anthology of results

[1] Acronym for *Education/Training/Teaching/Culture*. It is based on the former Science of Education Bachelor's degree.

[2] One of the teaching units in Year 1 is devoted to scientific culture.

surpassed expectations: both for the people (Archimedes, Plato, Aristotle, Egyptians, Babylonians, Pythagoras, Diophantine, Fibonacci, etc.) as for the period (from ancient Egypt to the 17th century) or places (Western or Eastern Europe, Middle East, China), everything seemed mixed up to the point of absurdity. My conclusion would be: telling math stories to young adults lacking historical landmarks is not always fruitful.

2.2 *Wingardium Leviosa*: the dangers of magic wands

The magic wands of teachers do not have the powers of Hogwarts wizards' wands. The first one is not rigid, but flexible: it is called the thirteen-knot-rope, or the arithmetic rope. For unexplained reasons, this modern myth has taken root in the teaching and teacher training worlds. It has even met there with some success, fueled by online documents which claim the historical reality of this object without great hindsight.

Let's first explain what this artifact is. It is a cord divided into twelve equal parts by thirteen regularly spaced knots. Both ends of the cord carry a knot. By superimposing these two knots, then stretching the string by the 5th and the 8th knots, you obtain a triangle whose sides measure 4, 3 and 5 equal intervals respectively. The Pythagorean Theorem ensures that the triangle is right-angled and consequently you have constructed a right angle!

This thirteen-knot rope gives substance to the theorem. This is probably what seduces the teachers (and others) who peddle this myth. The corresponding Wikipedia page ("Arithmetic Rope") shows that there are still unattended leaflets on that non-academic encyclopedia. Actually, the main illustration comes from a now missing manuscript in which it was captioned *Arithmetica* (Figure 2.1, truncated on Wikipedia).

Figure 2.1

But the original picture shows the use of a counting device for calculation with tokens. On the webpage it is also stated, without any serious documentation, that the thirteen-knot rope was widely used in Egypt on the building sites of the pyramids, and in the Middle Ages by cathedrals builders. This unverifiable piece of information is taken up by several French pedagogues on their personal websites.

Anyway, the wondrous aspect of this artifact makes it an educational attraction, since it materializes a Pythagorean triplet in a minimalist and elegant way. So why cast doubt on its existence? For a long time, many French IREM groups on history of mathematics have worked on practical geometry of the Middle Ages and Renaissance, up to the modern era. The list of perused books and manuscripts is very complete, even if it can't claim to be exhaustive. It turns out that so far none of the works studied by these groups for years mentions any knotted rope whatsoever, while the use of other instruments (false square, geometric square, astrolabe, ropes and stakes, etc.) is very frequently described. By the way, why would cathedral builders have bothered creating thirteen knots in a rope when only the fifth and eighth ones were used? Or that the use of removable flags or simple marks on the rope would have been equally effective? There are very old testimonies of the use of ropes for practical geometry, in Egypt by arpedonapts (literally "rope stretchers", the surveyors) or in the *Sulbasutras* in India (Sen & Bag, 1983). In the old texts that have survived, we find no traces of knots on the ropes, except possibly at their ends to attach them to stakes.

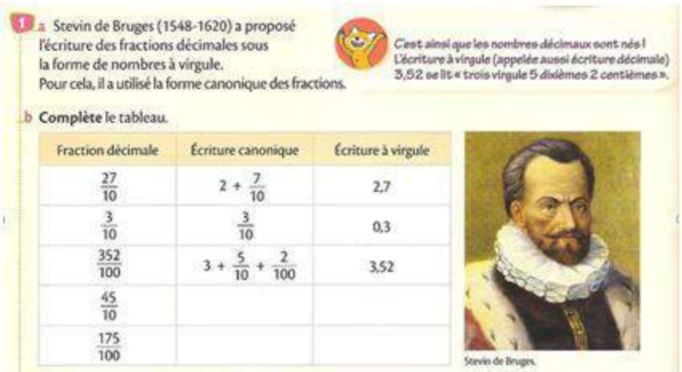
As a modern myth of the educational world, the knot rope does not require any philosophy of history, contrarily to the second magic wand, which is another theory that teachers also echo without precaution: the obscure role of the Golden ratio.

Once again, Egypt is the imaginary place of a mysterious science of the Great Old Ones. The fascination with the secrets of making pyramids began at the end of the 19th century with the publication of a study (Smyth, 1864) who founded pyramidology. One of the great finds of pyramidologists is the discreet presence of the Golden number (Φ) or π in the architecture itself. These values were obtained after playing with measures of certain lengths, even sometimes setting the value of the units arbitrarily. Many publications have shown the vacuity of such assumptions (Markowsky, 1992), but the golden number and its cosmic virtues retain some credibility among the general public.

At school, the Golden number usually arises when it comes to cross-curricular activities between mathematics and the arts. How many times have I heard that classical aesthetics is regulated by the divine proportion invented by Fibonacci but used long before in architecture? Almost always, secondary school teachers invoke a statistical truth: when you present to the general public rectangles of various shapes by asking them which one is the most "beautiful", the most often chosen rectangle would be the one whose dimensions are in proportion to the Golden number. This unfounded assertion has been denied more than once by actual polls (Jacquier & Drapel, 2005). Unfortunately it seems that Φ doesn't play any special role in beauty.

2.3 The dangers of handbooks inserts

Let's go back to Stevin's *Thiende*. No doubt about its popularity more than four centuries after its publication, since a recent French primary school handbook quotes it in its chapter on decimal fractions and decimal numbers (Briand & Peltier, 2016, 170). The sidebar is likely to be a pleasant illustration of the subject and to anchor Stevin's portrait in the students' imagination (Figure 2.2).



1 a Stevin de Bruges (1548-1620) a proposé l'écriture des fractions décimales sous la forme de nombres à virgule.
Pour cela, il a utilisé la forme canonique des fractions.

C'est ainsi que les nombres décimaux sont nés ! L'écriture à virgule (appelée aussi écriture décimale) 3,52 se lit « trois virgule 5 dixièmes 2 centièmes ».

b **Complète** le tableau.

Fraction décimale	Écriture canonique	Écriture à virgule
$\frac{27}{10}$	$2 + \frac{7}{10}$	2,7
$\frac{3}{10}$	$\frac{3}{10}$	0,3
$\frac{352}{100}$	$3 + \frac{5}{10} + \frac{2}{100}$	3,52
$\frac{45}{10}$		
$\frac{175}{100}$		

Stevin de Bruges.

Figure 2.2

Beyond aesthetics, the sidebar can useful by the link given between three parallel forms of decimals, very precisely found in the original text and justified through it. In this sense, it is an example of a very relevant use of the original resource. But it reveals some blunders. For example, you could be surprised by the choice of a fanciful portrait of Simon Stevin to illustrate the inset when there exists an engraved portrait of the author's life, which correctly mentions his identity (Simon Stevin) while he is presented in the box as “Stevin de Bruges” which might suggest to the students that this is a nobleman named Stevin. What is more annoying is that it can be read that Stevin “proposed writing fractions as numbers with commas” and that “that is how decimals are born!” which is incorrect. Finally the recommended spelling of number 3.52 in the form “three point five tenths two hundredths” seems inappropriate to us.

Since this type of sidebar is not uncommon in secondary school textbooks, it can be assumed that the new high school programs will generate new productions in response to their explicit references to history of mathematics. Let us hope that the contents will not only be chosen as nice images, and that they will generate activities based on original resources.

3 Beyond dangers: the use of original materials in the classrooms

The pitfalls I mentioned about introducing a historical perspective into the classroom might confuse and distract teachers from the use of ancient resources left to specialists in history of mathematics. As we have seen above, using ancient texts in class offers no secure way from the point of view of historical accuracy, but it would be a shame to deprive anyone of the light shed by the introduction of a historical perspective on mathematical notions. A multidisciplinary approach contributes to a better understanding of possibly tough concepts and it stimulates the connections between several fields of knowledge for the learners.

For the moment there are but few examples of using original texts in primary school mathematics. It is a recent but more and more frequent concern for teacher trainers, especially historians of mathematics who ponder on the implementation of primary school activities based on a reading of original sources. This is evidenced by the recently published *Passerelles* (Moyon & Tournès, 2018), which offers nine chapters on various approaches to mathematical concepts in reference to their story. The first activity paths that are presented below are in the same perspective as the chapters of the book. These activities have been mainly practiced in the last two years (2016-2018) in various classrooms, mostly with students of the Faculty of Education (ESPÉ) or elementary schools, as well as larger public or teachers during training courses or Popular Science events[3]. For activities at various levels there are many interesting HPM papers (Chorlay 2016, Métin 2012,).

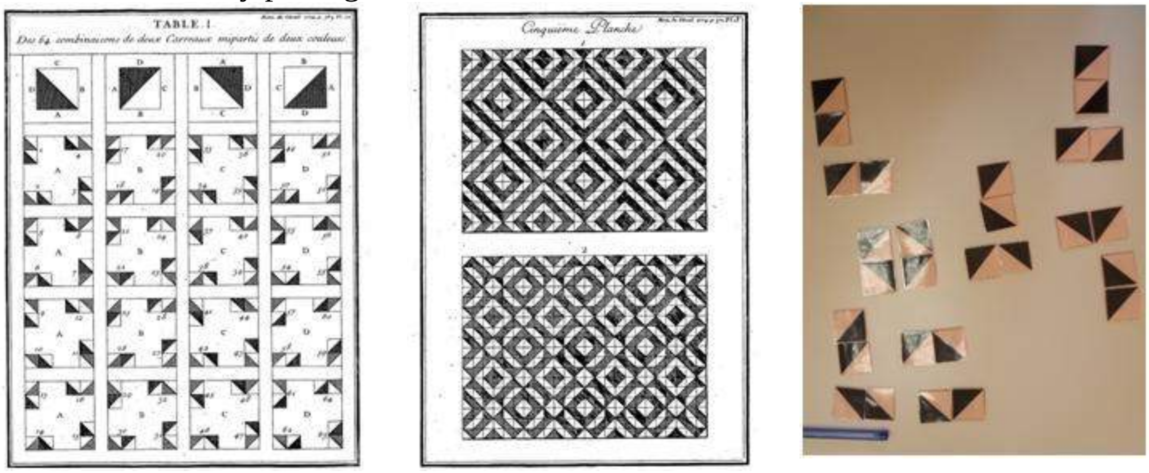
3.1 Floor tiling! Truchet’s patterns

Our first activity has been tested both in nursery school and teacher training. Readers will agree that it is difficult to undertake activities based on reading original documents for kindergarten. But it is possible to take the content of a text both as a resource for an activity (in kindergarten) and as a problem subject in itself (at the university), insofar as this text exhibits materials which can be manipulated even outside their original context. First the historical context of the document is set, and then its content is described. Finally I show the way I used it with children and university sophomores.

[3] For activities at various levels there are many available HPM papers (for instance Chorlay 2016 or Métin 2012).

3.1.1 Sébastien Truchet's Memoire

Father Truchet was born in Lyon in 1657. Naturally focused on mechanics, he was quickly noticed by the king's entourage for his clockmaking skills. He was appointed honorary member of the Academy of Sciences after having probably participated in many hydraulic works in the parks of the castles of Versailles and Marly. As he recounts in his *Memoire* presented to the Académie des Sciences (Truchet, 1704), it was during an inspection visit of the canals in the Orleans area that he became interested in pavements. He had been lodged in a domain whose owner had decided to renew the chapel floor using simple two-colored square tiles separated in two parts by their diagonal (Figure 3.1(a)). In his *Memoire*, Truchet searches for all possible combinations of two such tiles (Figure 3.1(b)), and then he deals with compositions that can be made by putting two such combinations next to each other.



(*a*) (*b*) (*c*)

Figure 3.1: Truchet's tiles

3.1.2 The use of the *Memoire* at University level

I found that this material was suitable for combinatorial exercises including experimentation. The interest of Truchet's two-colored tiles lies in their material aspect that allows manipulation like "let's do it and see what we get", instead of trying to think first. In fact, most undergraduate students would have been unable to determine all the possibilities of contact of two sides of tiles.

The group having been divided into four sub-groups, the students had a large number of two-colored squares of laminated cardboard. By assembling these squares in pairs, each group had to reconstruct one of the four columns of the original plate (Figure 3.1(a)), whose first cells only were displayed. So, the original material wasn't left untouched. On the contrary, the words had totally disappeared, as well as a part of the original plates.

What appeared clearly in this activity are the predominant places of visual observation and the faculty of manipulating objects. Furthermore the use of speech is at first counterproductive. For example, on the production of Figure 3.1(c), how can we determine the missing configurations? This question was a headache for the students of the group in question, who had to classify the configurations already present according to visual criteria, and name the classes obtained to be able to communicate with each other.

In the second part of this activity, the aim was to reduce the number of cases to its minimum by matching similar configurations. Students still used visual evidence, but it was sometimes difficult for them to identify the same configurations when in different positions. Note, however, that students tended to assemble tiles of the same color, as I gave indifferently black-and-white or orange-and-white tiles. I have not studied the impact of colors on students' success, as they did not initially plan to mix tiles of different colors.

3.1.3 The activity in kindergarten

Of course, there is no space for combinatorics in kindergarten. I nevertheless tried to use the cardboard material manufactured for the occasion. The proposed activity had to be supervised by students from the Faculty of Education. It consisted in the duplication of simple configurations inspired by those of Truchet's *Memoire*. The main learning goal of this activity was to develop the children's skills of spotting things in plane geometry without coordinate system, and determining relative positions of geometric shapes. The second objective was to lead pupils to feel the necessity and usefulness of an appropriate geometric descriptive vocabulary.

In some groups, there were harsh discussions because many pupils did not agree on the success of their partners even if they had difficulty explaining the mistakes. Generally speaking, the troubles came from the figures' orientation. University student helped the groups reconcile by leading configuration analysis and requesting expressions to characterize misplaced squares. The necessary manipulations were based on simple transformations: rotation and bilateral symmetry. It should be noted that two symmetrical assemblies were often taken as identical. This is perhaps one of the major disadvantages of two-colored tiles.

I am far from having exploited the full potential of these artifacts for the study of geometric shapes in the classroom. It is even possible outside the classroom, as two-colored tiles still exist today in real life; it is possible to buy them in DIY superstores, for example the "Dément" (that is: mentally ill…) collection in the French brand Leroy-Merlin. The website of this big brand even offers a visualization of some arrangements for bathrooms, in which I had the surprise to find the configurations already present in Truchet's *Memoire*.

No school so far has accepted to host a restoration of the soil of its classes, but the computer generated images of the aforesaid store website[4] let us hope that a virtual renovation project can be considered.

3.2 Counting with tokens

The second chapter of *Passerelles* (Moyon & Tournès, 2018) highlights the relevance of using token charts in primary school, in order to introduce pupils to the history of calculus techniques, from the practice using artifacts to the use of written signs. The authors set out the history of counting tables and their possible pedagogical use. The counting frames manufactured by Dominique Tournès and his IREM group are made of lines drawn on paper. The lines carry the tokens according to their order of units, tens, hundreds, etc. My practice of calculating tokens is quite different, as I draw it from books (esp. Anonymous, 1509 and Anonymous, 1551) which present the

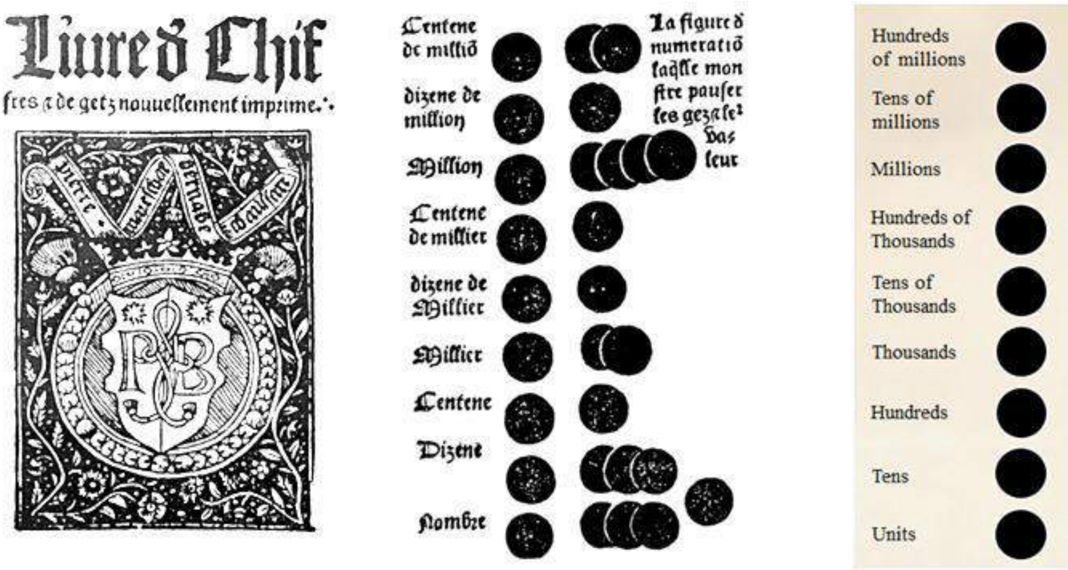
[4] https://www.leroymerlin.fr/v3/p/produits/carrelage-sol-et-mur-noir-blanc-effet-ciment-dement-l-20-x-l-20-cm-e1500488011. Accessed: 15 November 2018.

calculations without lines. Moreover, I do not use tokens with beginners in calculus, but with pupils, students and adults already familiar with the decimal position system and usual operations. Confronting these audiences with that forgotten practice allows them to revisit their representations of numeration and calculations.

It is often for pleasure when it comes to adults. However, it seems also conceivable to propose calculating tokens as remedial teaching in case of learning difficulties with counting and operations. As I will show, when you do not impose a particular technique, letting people appropriate the device, it allows them to question their personal representations of the decimal position system.

3.2.1 Resources

There are many arithmetic books printed at the beginning of the 16th century, mainly in Lyon, which at that time was a major center for both trade and printing. My main reference is a very rare book, named *Livre de Chiffres et de Getz*[5] (Anonymous, 1509), of which remains only one known copy, owned by the Méjanes Library of Aix-en-Provence. Its author is not identified, but the content is not entirely original since it is found partly in 15th century manuscripts and other 16th century printed works. The title page of the book bears only the identity of the printers, Pierre Mareschal and Barnabé Chaussard. As the printers's mark is damaged down on its right side, we can guess that the book was printed between 1508 and 1510 (i.e. when the association between the booksellers stopped), that is the reason why I chose to date it back to 1509.



(*a*) (*b*) (*c*)

Figure 3.2

Except for its rarity, the *Livre de Chiffres et de Getz* follows the same pattern as other books of its time. The first part explains how to manipulate tokens, for the representation of integers (*Numeration*) as well as the four operations. The second part deals with problem-solving based on the rule of three. In my classroom activities or during public sessions, I only present the title page and the caption entitled "The figure of numeration, which demonstrates how to set down the tokens and their value" (Figure 3.2(c)). The first

[5] Which means: *Book on Numerals and tokens*.

exercise consists of understanding how to put the tokens and find the value of the given number. Since no line is engraved on the board, the orders of the quantities are therefore indicated by a column of tokens, called “the tree”, to which the printers took care to add a legend that the practitioners of arithmetic obviously did not have at hand.

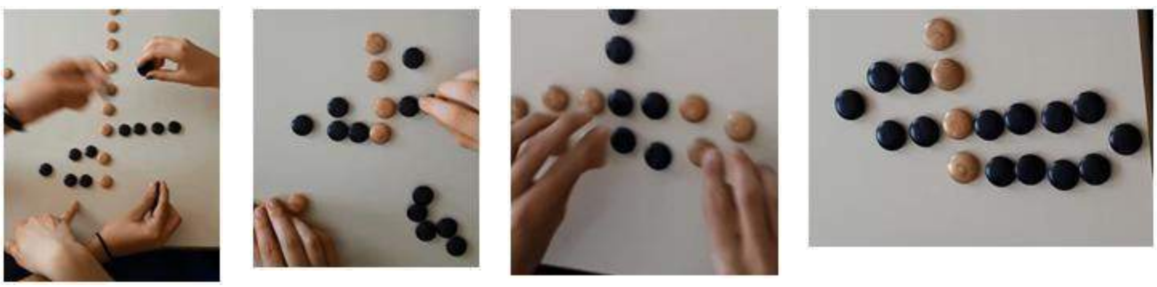
3.2.2 Manipulating tokens in the classroom (and elsewhere)

One of the greatest interests of this abacus without line lies in its very easy manufacturing and installing wherever wanted: just have a good amount of tokens (in my case, tokens from a game of draughts) and a flat surface. The participants in the activities themselves build their computational tree. In this respect, I advise beginners to have tokens of different colors, as can be seen on the pictures in Figure 3.2.

First of all, participants must find the value of the number given as an example in the book. They have but little difficulty in associating each group of tokens with the magnitude of the “branch” it belongs to, and thus to reconstruct the number (which is 214,112,138). Some numeration exercises are an opportunity to explain the role of the so called *quinary* token, placed in an intermediate position between two branches and representing five tokens of the lower branch. One possible explanation comes naturally: beyond five, there is little chance of being able to immediately perceive quantities without counting. In the context of monetary accounts, it would be easy for a virtuoso (but dishonest) arithmetician to take a token here and there and steal his customers. The quinary token allows all the protagonists a visual control of the operations.

Simple operations, namely addition and subtraction, do not need any explanation: in order to add any two numbers in the tree, all you have to do is cumulate the tokens of the same order and, when a sum reaches ten, to place a single token on the branch immediately above them.

When it comes to multiplication, the custom is to place the multiplicand on the left of the tree and the product on the right. The multiplier is not represented, and this is precisely the reason for the diversity of possible techniques. My first example is the multiplication by 20, often used in the old texts for conversions of money because a pound corresponds to twenty shillings (and a shilling to twelve pence). The task illustrated in Figure 3.3(a) is the conversion of 27 pounds (on the left) into shillings. The participants perform it very simply: each token on the left gives birth to two tokens on the right on the next branch up above. The quinary token will thus give two tokens between the tens and hundreds branches, replaced by a token on the hundreds branch. The main difference between the manipulators is their choice of leaving the multiplicand intact or, on the contrary, eliminating each token as it is replaced by the other two on the right.



(*a*) (*b*) (*c*) (*d*)

Figure 3.3

After this rather easy first exercise, I ask the participants to convert 27 shillings into pence, which is equivalent to multiply 27 by 12. Now, with this multiplier, we encounter different techniques. Let's describe three methods illustrated by Figures 3.3(b), 3.3(c) & 3.3(d) respectively:

- 3.3(b): Replacing each token on the left with twelve tokens on the right, that is two tokens in the same level branch and one token in the next branch up above. The quinary token makes no exception and it is only after placing all the tokens on the right that the simplification is carried out.
- 3.3(c): Number 27 is identically copied on the right, then all its tokens are raised by a branch ("we do: times ten"), finally each token on the left is doubled and moved to the right into the branch of the same order. The quinary token is simply raised to the branch of tens. The third pupil of the picture will find 374, because in his first raising, the quinary token was erroneously placed on the branch of the hundreds instead of being in the intermediate position.
- 3.3(d): The tokens on the left are reproduced on the right and doubled (but the student forgets to double the quinary token). Then, instead of raising the right stack of tokens from a branch, the students operate this move on the left tokens. Then they stop, puzzled (this is where the picture is taken). The strictly gestural aspect of the procedure led to a dead end. Resuming the procedure step by step, with discussions and comments, allows them to rectify the manipulation, by correcting the error on the quinary.

These various techniques are guided by different decompositions of multiplier 12, either as a block (Figure 3.3(b)), as 10 + 2 (Figure 3.3(c)), or as 2 + 10 (Figure 3.3(d)). The most interesting moment for the participants is the verification of their result and search for a possible error. For that purpose it is useful to film them after asking them to restart manipulations for the camera provided that they comment their actions, though it is not natural for them to speak while operating.

I have observed many times that this tokens calculus activity can be conducted without using language, because the essence of the practice lies in gestures which anyone can reproduce in front of other people without a word. It could almost happen in complete silence. However, I systematically ask for the explanation of the manipulations, whether the operators are beginners or experts, because without the verbalization the exchanges would only be visual and they would remain at a technical level. Since practice is not science, neglecting words would condemn participants to act only as performers.

3.3 Experiencing math oddity

Let's briefly mention classroom experiments based on arithmetic texts of the late Middle Ages and Renaissance. These texts do not use any algebraic formalism, but they expose problem solving algorithms without the use of letters. One of the works I have used the most is the *Oeuvre Tressubtile et Proufitable*[6] by Juan de Ortega, the first printed Spanish arithmetic (Ortega, 1515).

The simplest way to share the experience of math oddity with readers is to give them the original text of one of the reading / interpretation activities offered to high school students (Ortega, 1515, fol. XCIXr -our translation from French-)

[6] *Very Subtle and Profitable Work.*

A man makes his will and he has 3000 crowns left and he leaves his wife pregnant. And so he orders that if he dies and after his death his wife gives birth to a son, the said son will have the three parts of his property and the mother the other part. And [if] she gives birth to a daughter, the mother will have the three parts of his possessions and the daughter will have the other part. What happens is that after the death of the father, the woman made two children together, namely, son and daughter. It is asked how the possessions of the deceased person shall be divided so that the will of the father be observed.

This kind of problem is called "problems of will", and it is often found in old books on commercial arithmetic. The popularity of these problems may come from the need to turn to arithmeticians or even algebraists for inheritance disputes, because the laws on inheritance were extremely complicated for ordinary men.

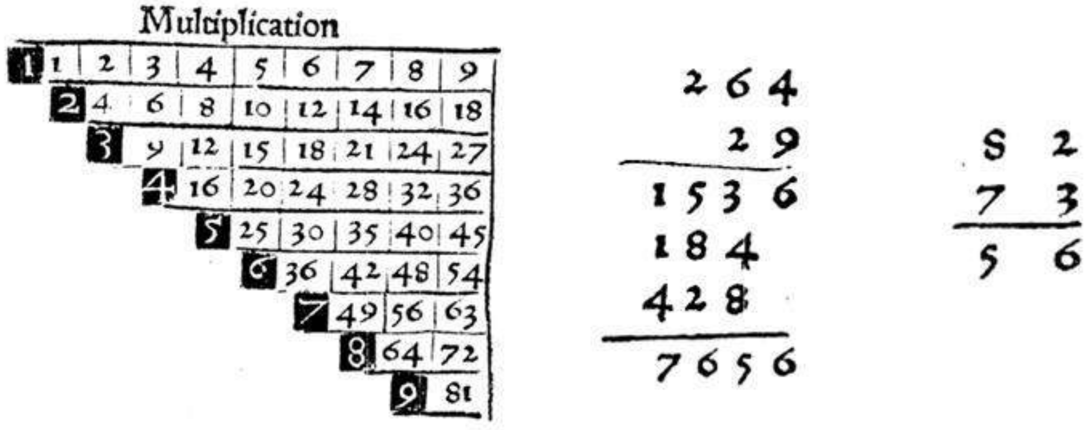
In the classrooms, the first obstacle to understanding comes from typography. It is however surprising to realize that generally no students are upset by the fate of the daughter. Anyway, once the language barrier has been overcome, understanding Ortega's solution can be quite uneasy:

And you will do that way: start with the daughter because if the daughter has a part the mother must have the three parts. For this pose 1 for the daughter and 3 for the mother, and the son must have three times as much as the mother and it will be 9. Now, add these three sums that is to say 1, 3, 9 & are 13 for the divider. Now, say by the rule of three: if 13 gives me 3000, what will give me 9? Multiply and divide as the rule of three requires & you will find 230 crowns & $\frac{10}{13}$*, which is the share of the daughter. And to the mother 692 crowns and* $\frac{4}{13}$*, and to the son 2076 crowns and* $\frac{12}{13}$ *of a crown.*

Mathematics teachers will quickly grasp that these are proportional shares, but you must remember that these notions are far from the concerns and skills of contemporary French high school students. These stumble particularly on the coefficients 1, 3, 9 and 13, the role of which in solving the problem they hardly understand. Here the numerators and the denominator are not distinguished by their positions in a fraction but by the word "divider", which qualifies the number 13.

It is certainly not the most difficult text that I have proposed to high school students. As a rule, the initial lack of understanding, frustrating as it may be, nevertheless leads to the satisfaction of perceiving the meaning of the texts through their mathematical contents before grasping the words. For us, the best work in this perspective remains Valentin Mennher's *Arithmetique Seconde* (Mennher, 1556), with its old fashioned algebraic resolutions of equations. For a more detailed study of that question, I will refer readers to another article (Métin, 2012).

Even without words, old practices in mathematics can turn out to be strange for contemporary readers. For instance, Figures 3.4(a) to 3.4(c) are visual excerpts from Robert Recorde's *Ground of Arts* (Record, 1543). Figure 3.4(a) shows a multiplication table that our contemporary students consider incomplete or halved. On Figure 3.4(b), the same students can't recognize their multiplication algorithm, but it is their algorithm, with the exception of the carried numbers. Figure 3.4(c) is the most puzzling. It shows the method promoted by Recorde to avoid learning the multiplication table for numbers over 5. For instance, if you want to know the product of 8 multiplied by 7, just evaluate the differences between each of these two numbers and 10: the “rests” are 2 and 3 respectively; firstly multiply 2 by 3 and you get 6; secondly calculate the difference between each initial number and the “rest” of the other one: 8 – 3, equal to 7 – 2, is 5. Concatenate the two results and you obtain 56, which is the result of the multiplication 8 × 7. Amazing! After this element of surprise, the students invariably go through two steps: How (does it work)? Why (does it work)?



(*a*) (*b*) (*c*)

Figure 3.4

To favor investigation and questioning, it is worth removing all words from the original excerpt, leaving only the operations. The surprise comes from the lack of information (“I don’t know the reason why it works”), so don’t provide the students with too much information. Even a simple picture can sometimes be useful, as I show now.

3.4 Back to Louis XIV’s Versailles in primary school

At primary school level, the history curriculum gives prominence to great French figures of the past. Many aspects of the Grand siècle can give birth to original activities about symmetry or geometrical construction programs. In the context of a lesson study workshop, I had to find documents adapted to young pupils who are in the process of discovering symmetry. As my homeland of Dijon is renowned for its culinary culture, I was prone to look for them in the feasts.

3.4.1 The initial document

An impressive table plan is displayed in the kitchens of Chantilly castle, which were the quarters of the famous cook Vatel (Figure 3.5). This is the plan of a table for King Louis XIV at the castle of Marly-le-Roi, as it had been submitted to him in 1699. At Marly, the king took his meals with the royal family and particularly distinguished courtiers. The single rectangular table had been replaced by two oval tables and the plan shows the layout chosen on a special occasion. This arrangement was surely guided by symmetry, not complete however since there are seventeen covers. Perhaps symmetry was not desirable? The king having no counterpart, he could only sit in a unique, distinguishable place, which may correspond to the isolated cover at the bottom center of the ellipse.

The arrangement of dishes on the table is part of the aesthetics of the whole. The plates of the guests are represented by disks distributed around the entire circumference of the oval. At the center, a curvilinear hexagon must correspond to a centerpiece carrying spices and herbs. Between plates and centerpiece, the “Roast dishes” materialize an axis of quasi-horizontal symmetry, while the circular soup tureens and the “Pots-à-oille”[7] are arranged in a symmetrical way compared to the central one. The round plates of hors d'oeuvre complete the set to occupy a maximum of space on the table.

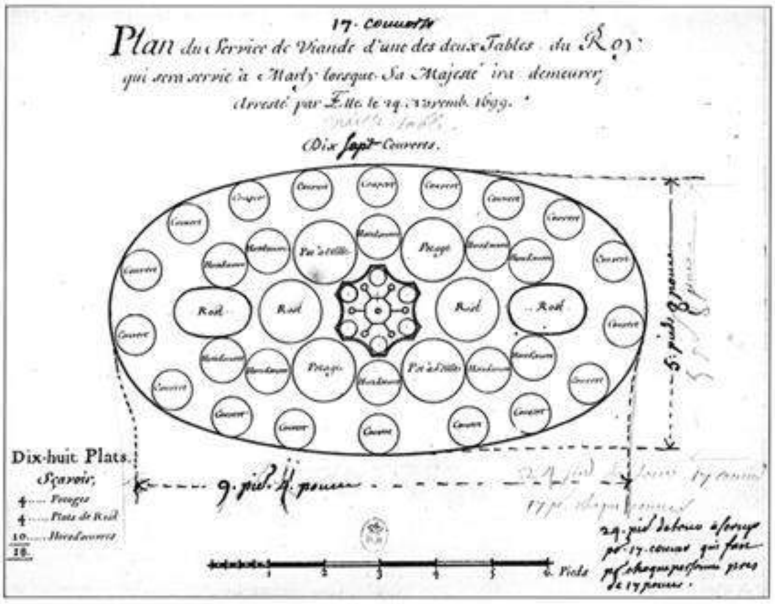


Figure 3.5

This document, strongly linked to the history of France (as far as butlers are concerned), was a useful starting point for discovery and deepening activities on symmetry in primary school.

3.4.2 Activities in the classroom

The colleague who welcomed me in his classroom was a highly qualified user of interactive whiteboard and computer technologies in general. After a short time of exchange on the historical context of Versailles in the time of the sun-king, the table plan of 1699 was dissected, insisting on the organization of the reception and the placement of the guests and dishes on the table. When a first “mirrored” idea of placement was formulated, the pupils were invited in turn to make this property appear by coloring: a first pupil chose an object which they painted, the next one must use the same color to paint one of the objects that corresponds to it and the result was submitted to the entire class.

Introducing the notion of axis of symmetry was facilitated by the interactive coloring task and the final request: “Where could you cut this table to obtain two exactly identical parts?”

The plenary activity generated but few errors as the exchanges within the class were successful and the incorrect proposals were immediately corrected. However it should be noted that a second axis of symmetry, this one being horizontal, had been accepted by all (including the teachers) on the basis of visual evidence, while the number of guests around the table did not allow it. It would have been necessary to agree on a partial symmetry, but

[7] These tureens were both containers for meat in sauce and prestigious elements of decoration.

the adults in the classroom preferred not to go too far with this notion which could cause some trouble among the pupils.

Further activities have consisted in reconstituting incomplete figures by symmetry, all the symmetric objects chosen according to the royal theme: chandeliers, gardens, windows, facades, and so on. I even launched a more complicated activity on napkin folding, which proved to be as promising as this new field of research in history of mathematics (Friedmann & Rougetet, 2017). But to keep the promises, I still have to find a better way to use it in the classroom.

In a simpler follow-up activity, I had the pupils reconstruct the king's table of which they had only one half and the tracing paper of the other half to color symmetrically. Some of them adopted a simplifying strategy of returning their tracing paper, reproducing identically the image given on the tablet, and finally restore the initial orientation of their paper. This strategy was clever in this case because I hadn't been careful of creating table layouts with only one axis of symmetry. In addition, the pupils' ability to estimate the success of their work at a glance made all point-by-point examinations useless. I had to find a trick to force them to analyze their production.

Thus I decided to create brigades of waiters, butlers and wine waiters: the pupils would set the table for a royal banquet. Lacking porcelain and silver tableware, they prepared half-tables with plates and plastic cups, placemats and paper napkins, aluminum dishes, all usual school material. A red tube from the gym serving as an axis of symmetry, the brigades were ready to set the table. Given the length of the table (about ten meters) and the height of the pupils, it was not possible for them to supervise the work from above, especially since it was a collective task evaluated by a single Controller of the *Menus-plaisirs* of the king.

By themselves, pupils tend to perform the task in two stages, successively considering the position and then the orientation of the objects to be placed. They first assign a location to the various elements. Only in a second time they pay attention to the orientation of these objects, which generates discussions and even a contestation of the authority of the Controller. Some pupils consider that the first stage was sufficient for the task to be completed, thus focusing on the functional aspect of cutlery and dishes, while the aesthetic aspect was emphasized by the verifiers.

4 Texts as artefacts

We may consider old book as objects as well as sources of knowledge. As in most of school topics, many teachers and students refer to the written word as inarguable. So when you propose a text for reading, it is always taken as the honest truth. Fortunately, there are at least two manners of contradicting that postulate.

First, you can choose obscure writers from the past, as I did with Juan de Ortega, Samuel Marolois or Jean Bullant (Métin, 2006) whose names and works are unknown to our French students. There are so many now forgotten texts, that we have a variety of true errors and mistakes close at hand.

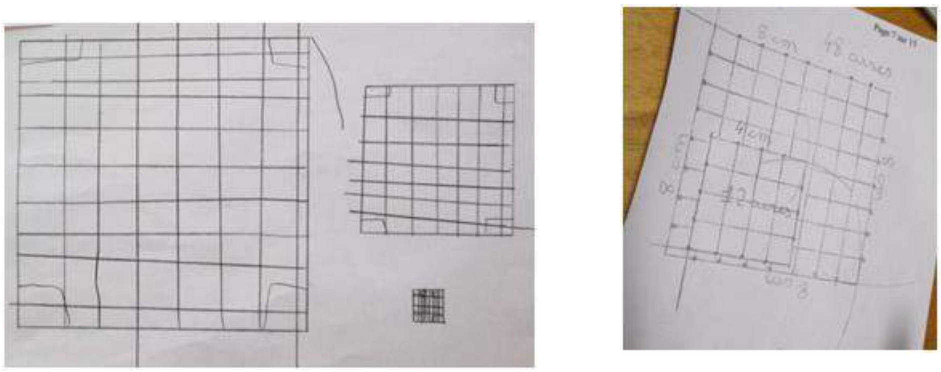
Secondly, you can partly remove texts or images from the original pages, even when the texts are perfectly understandable by themselves. For instance in 17th century books on geometry, the names of points were often displayed before these points were actually defined. The readers were then supposed to follow the construction steps on fold-up plates. The complete figure being under the reader's eyes, the points were used both as mathematical points and as parts of an image. This way of describing figures before defining their components was also in use in manuscripts, whose writers sometimes set drawings apart. When the plates or illustrations are lost, then our task is to reconstitute the whole. But as this reconstitution leads us to dig deeper in the subjects, and offers the opportunity of an exciting investigation, why not provoke the absence?

4.1 Doubling the square with or without Plato

In Plato's famous dialogue Meno, when Socrates puts forward the duplication of the square, he does it to support his theory of reminiscence. The dialogue was the subject of an interdisciplinary mathematical / French research conducted by a Paris IREM team, and written by Renaud Chorlay as the fifth chapter of *Passerelles* (Moyon & Tournès, 2018). There are no figure dating back from the ancient times in the dialogue, it is not even sure that they might have existed at that time. The figures have for a long time been reconstructed by Plato's readers, but it is an exciting exercise to try to understand the text without them and to reconstruct them in the course of the reading. Despite the difficulties of the text, the chapter highlights the interest of the activity in French as in mathematics. In the Paris IREM group project, words and speech are at the center of learning objectives.

It turns out that I had also worked on the duplication of the square, but in a different perspective. After discovering fractions, pupils in an elementary school in Dijon had troubles with dividing quantities. The very nature of fractions didn’t go without saying. I therefore took advantage of this moment of doubt to suggest a “little exercise”, namely the construction of a square whose area was double the area of a given square.

As in the Paris experiment and as in Plato’s dialogue, the pupils’ first reaction was to double the side of the square, either numerically from its measurement, or by using the compass. How do you explain them that the area of the square they obtained is not twice the original area? They found by themselves a way of testing their solutions in subdividing each of the obtained squares into squares of equal or nearly equal areas (see Figure 4.1(a)), a process they called “areas technique”.



(*a*) (*b*)

Figure 4.1

When the subdivision was accomplished according to the standards, it is the calculation that left something to be desired (Figure 4.1(b)): the preteen here considered that a square of side 4 had a “12 areas” area and that its double, whose side is 8, had an area of “48 areas”. He could not explain his calculations, but he was happy with his result since, like his friends, he had obtained four times the area of the initial square!

Where were we, between text and artefact? I must say that no word by Plato was ever revealed to the pupils. Nevertheless, their own research directions followed the pattern of Meno's slave's ones. Of course, the teacher played the part of a modern Socrates, questioning preteens in the right direction. Meno's initial square had served has an artefact source for a research activity. Here I had removed the text, but other experiments lie on the removal of pictures.

4.2 Removing pictures

Our main research field in history of mathematics is 17th century military architecture. Of course, fortification is strongly linked to geometry, as it is a matter of lines and angles. But constructions are made under constraints and early modern fortifiers had to be able to compute the measures of lengths and angles, which implied a solid knowledge in Euclidean geometry.

Even for pupils without great knowledge of Euclidean theorems, it is possible to propose mathematical activities based on old fortification treatises. Readers will find such activities in my other article in the present volume. The basic activity focuses on drawing the shape of a polygonal fortress according to the construction programs we find in original sources, as for instance *La Fortification reduicte en art et demonstree* by Jean Errard of Bar-le-Duc (Errard, 1620).

During my researches on military architecture, I discovered a very special handbook written by a mathematics teacher of a French Artillery Academy (Famuel, 1684). What was special in this book was an almost white page, with an empty space facing the detailed construction program of the fortified hexagon. I realized that all illustrations in this handbook were handmade. It was used by his author as a pedagogical support: the students were given (or certainly must buy) the booklet with the complete text, but no illustrations at all, and they had to complete the empty spaces with the figures corresponding to the construction programs. It seems to be a trend in French Artillery Academies at the end of the 17th century, because I found two other books of the same type from the same period and the same context.

The natural challenge for use was to reconstitute the figure. As a teacher, I suggested my students to do it themselves, in order to complete the copy in the public library. This was such a funny and interesting activity that I decided from that day on to systematically remove pictures from fortification books. I already have mentioned the supplementary obstacle for contemporary students to follow instructions quoting names of points not yet defined. The advantage here is that the teacher from the past has created the text with full knowledge of his students' blindness. As the original pages were intended as supports for drawing, I do the same now, turning the geometrical books of the past into a kind of modern coloring book (without colors, just lines).

In case of dissatisfaction due to the lack of colors, you can turn to Oliver Byrne's edition of Euclid (Byrne, 1847). If you do so, your task will be to remove the colors from both text and illustrations, and to invite students to reconstitute the colored illustrations along with the colored parts of the texts. It appears that this is not easier than reconstruct missing black-and-white figures in 17th century fortification books.

4.3 Mathematics in the margins

Our last work lead about the use of original sources in the classroom deals with teacher training. I have used written resources as writing artifacts, that is to say, as objects designed to serve as supports for written commentaries about the texts they carry. It was not for students to undertake a textual exegesis of obscure passages of ancient texts, but rather to question the mechanisms they use to understand the contents of these texts.

Writing in the margins when studying a book is a well-established tradition in mathematics. Let's mention at least the most famous example: Pierre de Fermat annotating his copy of Diophantine's Arithmetic and leaving a marginal mention that would become his famous conjecture. According to Fermat himself the margin was too small for his clever demonstration. During my wanderings in libraries, I discovered numerous marginal notes, certainly of smaller impact than Fermat's ones, but witnessing the real interactive aspect of traditional books made of paper. In addition, whether written by well-known scholars or obscure readers, marginal mentions are manifestations of mathematics as human activities.

One of my latest research projects at Dijon IREM deals with written traces of student activity. In its didactical part this project focuses on the mathematics rough book seen as a private journal for students. In its academic part, it relies on the marginal annotation of original texts facsimiles, ranging from commercial arithmetic to algebra, geometry and probabilities. My analysis of the productions is centered on three lines of research: the necessity of rewriting or not, the need for translation in familiar terms or usual notions, and the place of the diagrams.

During University teacher training sessions, I propose students to work on photocopies, with instructions to jot down in their margins anything they had in mind (questions, reflections, astonishment) which they considered part of their learning mechanism.

I gave for example this extract of a curious book on practical arithmetic (Cathalan, 1566):

> *A man has 3 windmills, one of which grinds 5 bushels a day, while the second one grinds 7 bushels, and the third one 8. Comes the seller who wants to grind 100 bushels of wheat. I ask: how should the miller divide wheat between the windmills, so that all three windmills take the same time to complete the task?*

It is not that easy for young students to understand that a bushel is a measure of capacity (of eight gallons). Moreover, the French text mentions a specific measure, the *setier*; or "septier" as Cathalan writes it, which is a mystery to them. They finally hardly understand that this is a proportional sharing problem.

This is probably the reasons why students need to rewrite the text. Unsurprisingly, the words are a first obstacle, and may need a complete reformulation of the text, even if some students neglect the crucial point of the question, the fact that the three mills must have finished at the same time. Productions often show the students' efforts to stress the meaning of words, even if the text is not rewritten. Students use the photocopies to highlight words and redefine them. We must not be surprised by some students' lack of understandings. I purposely indicated that it was not a matter of perfect understanding, but of writing down all the elements of their search.

5 Conclusion

I have presented various activities related to the use of original texts in the classroom with the concern not to sacrifice rigor on the altar of the beauty of tales. Focusing on artefacts effectively saves us from controlling the historical context and precise circumstances. This could be summed up by this formula: not let oneself be locked in a historical horse collar when resources are relevant to mathematics, but allow oneself to explore the historical and social context of these resources. Let us try to summarize the lessons drawn from the various experiments I described.

5.1 What is doing history of mathematics?

In my opinion, doing history of mathematics is first of all practicing mathematics, provided that these mathematical activities are inspired by original sources that can be directly exploited with students or not. Therefore, it is not a matter of strictly historical methods. The questions are mainly aimed at understanding the concepts, even if the process of understanding includes domains off the beaten scientific track. However, calculating with tokens without questioning the medieval period and the link between objects and numbers would be a really dry activity.

In the case of the tiles, there is such a small difference between 18th century objects and the contemporary model that it was quite possible not to refer to the Truchet's text for the activity. However, the search for combinations of tiles is legitimized by the Memoire of 1704, whereas it could seem artificial or useless if it was simply asked as a research subject. This combinatorial exercise will be easily presented as a reconstruction of an ancient text, the existence of which generates verifications and comparison with the students' solutions.

5.2 An epistemological *e(n)strangement*

The concept of an epistemological *dépaysement* was recently re-presented by David Guillemette (Guillemette, 2015*a* & 2015*b*). What is at work here is the transformation of a familiar object (mathematical knowledge) into another less familiar object. The history of mathematics disrupts the usual view of mathematics by confronting us with writings and practices in which we do not recognize our own knowledge. What ordinarily went without saying becomes surprising.

In the examples I gave, the study of arithmetic texts of the Renaissance best allows the epistemological *dépaysement* of high school students and in training teachers. A major interest of the change of scenery here is to allow questioning new questioning by placing student in the dark in front of a text they do not understand at first glance. It happens then a triple effect for people in this situation:

1° as students are forced to deepen their look into matters they do not appreciate much they can first say "I do not understand a word!"

2° attempts to decode mathematics behind this foreign language leads them to reassure themselves about their own capacity for comprehension. Finally they discover they are able to interpret ancient languages (algebra) and methods; it is therefore a work of translation and commentary, because students have to insert complementary algebraic lines to take full advantage of the contents.

3° after being relieved, students will have to think about the mechanisms that allowed them to rediscover the hidden mathematical notions. In fact, this understanding has been favored by the X-rays of their understanding, which showed them the mathematical skeleton of the text.

The *dépaysement* only makes sense if people change their views. As with travels, this change of view is due to meeting others. In other words, looking for the initial context of a mathematical notion or appropriating practices from another time brings us back to the experience of the unfamiliar. More than a change of scenery, it is a revolution. I had no words for this special concept, so I decided to create the neologism *enstrangement* to significate the ability of making strange what was familiar.

5.3 A non-magisterial context

Using original documents means that teachers have to accept not to be the unique source of knowledge. In fact, neglecting historical references leads teachers to presenting subjects as their own knowledge without mention of any third party. However it is possible to insert mathematics in the history of human progress.

In the case of Truchet's tiles for example, it is pleasant to offer students the opportunity to solve a problem already studied in the 18th century but still relevant and within their reach. Would they be interested in working on tile arrangements without the challenge of solving a question posed by an Academician of the Enlightenment? It is Truchet himself who asks the question to the students. This enquiry is a form of adventure that would not be allowed with the teacher alone.

Teachers must even accept to become simple guides of their students in front of a research put by a third person. No longer the exclusive owners of knowledge, teachers become travel organizers, facilitators of discovery, as they are when they take their students to museums. In this case, mathematical activity can't be reduced to problem solving. What's more, students here can both use reasoning skills and non-brain skills to experiment end manipulate objects. Working with artefacts is a matter of gesture as well as reasoning; you can't limit the scope of authorized skills, or you would restrict your students' abilities, even in the case of a text study. Favoring the surprise (the How? that leads to the Why?) implies making space for students to choose all available means to succeed in the task. But what may not be usual in mathematics (at least in France) is that we are thus led to leave space for students' expression.

5.4 Personal expression

It seems important to me to establish a dialogue with students about the studied mathematical concepts. Learners are well aware that they don't invent the properties of objects or the techniques and methods they discover in their activities. But one of the major difficulties for novice teachers is to listen to what students have to say about what they are studying, and even to encourage questions. In scientific activities, these teachers tend to provide all the answers and do not favor the emergence of questions that could deviate from the already marked path. This often happens to ensure the success of the activity.

In this case, teachers promote knowledge more than learning in general, and particularly methods. My proposal for an analysis of understanding based on marginal annotations of the texts is very recent and my working IREM group is in its early stages. I hope my young fellow teachers will agree with the need to promote a personal verbalization of their own students' path in the appropriation of methods or mathematical practices. By putting my trainee students in the position of exegetes, I try to help them to move away from the model of knowledge holders, an all-powerful posture linked to the status given by diplomas.

My goal would be achieved if I could find with them the pleasure of a Joyful Wisdom and the adventure of the search for knowledge. For me, introducing a historical perspective through the use of ancient resources, texts and artefacts, is a means of achieving this goal.

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