WHITEHEAD'S PHILOSOPHY OF MATHEMATICS AND EDUCATION AS A FOUNDATION OF DIALOGICAL TEACHING

Arto MUTANEN
Finnish Naval Academy & Finnish National Defence University, Suomenlinna, Helsinki, FINLAND
arto.mutanen@gmail.com

In the education of mathematics, and also in education more generally, a dialogical approach has been emphasized for several years (Hofmann & Ruthven 2018; Mercer, Dawes & Kleine Staarman 2009; Radford 2011; Koterwas, Dwojak-Matras, & Kalinowska 2021, Prottas 2018). However, dialogical teaching has long historical roots, as the Socratic Questioning Method shows. The examples of Bacon and Kant demonstrate that a rational questioning method must be based on some presuppositions, showing the methodical foundation of dialogical teaching. (Hintikka 2007.)

The dialogical methods of teaching emphasize the role of language in general (Radford 2011), the role of symbolism (Kitcher 1986), and reasoning (di Toffoli 2021), which is deeply connected to the philosophy of mathematics (Korhonen 2013; Hintikka 1973). Hence, it is illustrative to more closely consider A. N. Whitehead's ideas of the education of mathematics. Whitehead, in *Principia Mathematica* (written together with Russell), explicated the logicist philosophy of mathematics. The book was intended as a sequel to Russell's *Principles of Mathematics* (1903), but it "became increasingly evident that the subject is a very much larger one" than the authors had supposed.

A main intention in the logicist program was to reduce mathematics to logic (Steiner 2006; Benacerraf & Putnam 1964). Whitehead and Russell followed the logicist project, which originates from Frege's and to Peano's ideas. Besides logicism (Frege, Russell, Whitehead), two other main approaches in the philosophy of mathematics, namely intuitionism (Brower) and formalism (Hilbert), can be identified. The huge task of the *Principia Mathematica* was to make the reduction explicit (Steiner 2006; Benacerraf & Putnam 1964). So, it is not a surprise that the Principia became such an extensive work of three large volumes.

The *Principia* is a detailed analysis of mathematical concept formation and mathematical reasoning, in which logico-mathematical rigor was emphasized. The foundational ideas of it can be rooted into the Kantian idea of intuition (Anschauung), which is related to perception. Hence, intuition in mathematics is closely connected to the Kantian intuition-based epistemic approach (Korhonen 2013), which entails an important parallelism between the learning of mathematics and the learning of science (Hintikka 2007). The whole picture changed in the 1930s because mathematics was proved to be both incomplete (Gödel 1931) and undecidable (Turing 1936). The epistemological questions, such as questions about mathematical practice, became especially central in the 1960s (Murawski 2010; Korhonen 2013; Kitcher 1986).

In the study of the education of mathematics, the epistemological questions have been central ones which implies that the leading philosophy has been a gathering of different philosophies of mathematics (Steiner 2006). Of course, the logicist philosophy of mathematics has been of extreme importance for educators of mathematics, especially, for instructionists who emphasize, not toward learning but toward "compartmentalized and decontextualized facts", which are intended to be accomplished by clear and effective instructional techniques. There is no reason to believe that such education produces deep conceptual understanding (Sawyer 2014, 2-3), which was emphasized by Whitehead (1911; 1929). According to Whitehead without understanding, mathematics is only an aggregate of unintelligible theorems, which is "fatal in education". The conclusion is that "mathematics, if it is to be used in general education, must be subjected to a rigorous process of selection and adaptation", which entails that, in the education of mathematics, we have to "deal directly and simply with a few general ideas of far-reaching importance" (Whitehead 1929). Whitehead (1911) emphasizes that formal character allows us to extend "the number of important operations which we can perform without thinking about them", which is an essential step in cultivating "the habit of thinking of what we are doing". The formalism makes reasoning visible (de Toffoli 2021). Whitehead emphasizes the role of historical and philosophical knowledge in understanding the true character of mathematical knowledge, as in the case of geometry or trigonometry (Whitehead 1911). A central idea of Whitehead's philosophy of mathematics is the possibility to explicate semantical ideas behind formalisms, which are used as presuppositions of questions in dialogical education. The formal structure of the textbooks of mathematics

(Steiner 2006) entails the important distinction between procedural and conceptual knowledge (Hiebert & Lefevre 1986), which are closely connected: there is no procedural knowledge without some conceptual knowledge and vice versa. In education sciences, it is usual to study dialogical methods empirically (Hofmann & Ruthven 2018). There is also need for philosophico-conceptual study of strategic dialogical teaching of mathematics. Whitehead (1929; 1-2) emphasizes that the "ideas that are merely received into the mind without being utilized, or tested, or thrown into fresh combination" cannot be useful; Whitehead refers to such ideas as *inert ideas*. Socratic dialogues enliven mathematical knowledge. Whitehead does not refer to Socrates nor to Plato. But he has, in his mind, the method of analysis and synthesis, which has the same roots (Niiniluoto 2018). The method of analysis and synthesis can be seen as the foundation of the dialogical method, which can also be seen from the oral tradition, when oral dialogue was enriched by drawings in the sand (Radford 2011), which takes place in Plato's *Meno*. Plato argues that written language ends proper dialogue: the text answers questions always in similar words, which stops the dialogue. (Radford 2011.) Whitehead's idea of the foundational role of historical and philosophical knowledge for learning mathematics allows us to overcome the tension between the formal and substantial understandings of mathematics (Whitehead 1929), which allow us to develop dialogical methods of teaching and learning.

REFERENCES

Benacerraf, P. & Putnam, H. (eds.) (1964). Philosophy of mathematics: Selected Readings, Cambridge: Cambridge University Press

Davis, M. (ed.) (1993). The Undecidable. New York: Dover Publications.

De Tofoli, S. (2021). Reconciling Rigor and Intuition, Erkenntnis (2021) 86:1783–1802 https://doi.org/10.1007/s10670-020-00280-x

Gödel, K, (1931/1993). On Formally Undecidable Propositions of the Principia Mathematica and Related Systems. I. In Davis (1993), (p. 5-38).

Hiebert, J. & Lefevre, P. (1986). Conceptual and procedural Knowledge in Mathematics. In Hiebert, J. (1986). Conceptual and procedural Knowledge: The Case of Mathematics (p. 1-27). Lawrence Erlbaum Associates.

Hintikka, J. (1973). *Logic, Language Games, and Information: Kantian Themes in the Philosophy of Logic*. Clarendon Press.

Hintikka, J. (2007). *Socratic Epistemology.* New York: Cambridge University Press

Hofmann, R. & Ruthven, K. (2018). Operational, interpersonal, discussional and ideational dimensions of classroom norms for dialogic practice in school mathematics. British Educational Research Journal Vol. 44, No. 3, June 2018, pp. 496–514 DOI: 10.1002/berj.3444.

Kitcher, P. (1986). Mathematical Change and Scientific Change. I T. Tymoczko, (ed.), *New Directions in the Philosophy of Mathematics* (p. 215-242). Birkhäuser

Korhonen, A. (2013). Logic as Universal Science: Russell's Early Logicism and its Philosophical Context, Palgrave Macmillan; London; https://doi-org.libproxy.helsinki.fi/10.1057/9781137304858

Koterwas A, Dwojak-Matras A, & Kalinowska K. (2021). Dialogical teaching of research integrity: an overview of selected methods. FACETS 6: 2138–2154. doi:10.1139/facets-2021-0045.

Mercer, N., Dawes, L. & Kleine Staarman, J. (2009). Dialogic teaching in the primary science classroom, Language and Education, 23:4, 353-369, DOI: 10.1080/09500780902954273

Murawski, R. (2010). Essays in the Philosophy and History of Logic and Mathematics, Brill

Niiniluoto, I. (2018). Truth-Seeking by Abduction. Springer

Prottas, N. (2018). Between Practice and Theory Dialogical Teaching and Art as Performative, Museum Worlds: Advances in Research 6 (2018): 60–73; doi:10.3167/armw.2018.060106

Radford, L., (2011). Dialogism in Absentia or the language of Mathematics, in Silvia Sbaragli (ed.), *La Mathematica e la sua Didattica Quarant'anni di Impegno; Mathematics and its Didactics: Forty Years of Commitment. In Occasion of the 65 Years of Bruno D'Amore*. Pitagora Editirice Bologna

Russell, B. (1903). *The Principles of Mathematics*. Merchant Books.

Sawyer, K. (Ed.) (2014). The Cambridge Handbook of the Learning Sciences.2nd ed. DOI: https://doi.org/10.1017/CBO9781139519526.

Steiner, Mark, 2006, Teaching Elementary Arithmetic through Applications, in Curren, Randall (ed.), A Companion to the Philosophy of Education, Blackwell

Turing, A. (1936/1993). On Computable Numbers, with an Application to the Entscheidungsproblem. In Davis (1993), pp. 116-154.

Whitehead, A. N. (1911/2005). Introduction to Mathematics, New York: Barnes & Noble Books

Whitehead, A. N. (1929). The Aims of Educations and Other Essays. New York: MacMillan Company

Whitehead, A. N. & Russell, B. (2010). Principia Mathematics, Merchant Books.